Thermo-mechanical analysis of energy pile groups via the equivalent pier method

Introduction

Consider the group of energy piles of 20 m in length and 0.8 m in diameter that is reported in Figure 1 and has already been analysed previously. Remember that the energy piles are socketed in a saturated sand deposit and that a 12×12 m rigid slab (resting on the ground) made of reinforced concrete connects all the energy piles. The sand and the pile proprieties are reported in Table 1 and Table 2, respectively.

Assume that the behaviour of the considered energy pile group can be analysed with accuracy with the equivalent pier method, i.e., by modelling the pile group as a single equivalent pier.

For the considered pier, calculate the parameters needed for its geometrical and material description, i.e., the equivalent diameter, D_{eq}, the equivalent Young's modulus, E_{eq}, and the equivalent linear thermal expansion coefficient of the pier, α_{eq}. When calculating α_{eq}, assume that $X = \alpha_{soil}/\alpha_{EP} \leq 1$, where α_{soil} and α_{EP} are the linear thermal expansion coefficients of the soil and energy piles, respectively.

With reference to the bearing capacity of one of the energy piles in the group that has been previously calculated, determine the bearing capacity of the equivalent pier by distributing the total shaft and base capacities of the group (calculated as the shaft and base capacities of the single isolated energy piles multiplied by the number of piles in the group for hypothesis) on the shaft and base area of the equivalent pier, respectively. This implies that

$$q_{s,eq} = q_s \frac{D}{D_{eq}} n_{EP}$$

and

$$q_{b,eq} = q_b \frac{D^2}{D_{eq}^2} n_{EP}$$

To construct the load-transfer relationships for the shaft and base of the equivalent pier, consider that it can be reproduced by a revision of the relationships proposed by Frank et al. (1991) for piles in coarse-grained soils, i.e.,

$$K_{s,eq} = 0.8 \frac{E_M}{D} \zeta$$

$$K_{b,eq} = 4.8 \frac{E_M}{D} \zeta$$

where

$$\zeta = \frac{s}{L}$$

for which s is the centre-to-centre spacing between the piles and L is the pile length.

By using the software Thermo-Pile (Knellwolf et al. 2011) evaluate the average vertical displacement of the equivalent pier with depth in five different cases:

- CASE 1: pier free at the head subjected to a vertical load of $P = 4500$ kN and to a temperature change of $\Delta T = 0$ °C.
- CASE 2: pier free at the head subjected to a vertical load of $P = 0$ kN and to a temperature change of $\Delta T = 10$°C.
- CASE (1+2): pier assumed to be characterised by the effects induced by the loads considered in CASE 1 and CASE 2 through the elastic superposition principle.
- CASE 3: pier free at the head subjected to a vertical load of $P = 4500$ kN and to a temperature change of $\Delta T = 10$ °C.
- CASE 4: pier restrained at the head by the presence of the slab and subjected to a vertical load of $P = 4500$ kN and to a temperature change of $\Delta T = 10$ °C. Assume that the slab stiffness can be estimated through the following equation, with reference to a rigid rectangular plate resting vertically loaded on an isotropic elastic half-space (Gorbunov-Posadov and Serebrjanyi 1961):

$$K_{slab} = \frac{E_{soil}\sqrt{B_{slab}L_{slab}}}{(1 - \upsilon_{soil}{}^2)\rho_0}$$

where E_{soil} is the Young's modulus of the soil, B_{slab} and L_{slab} are the dimensions of the slab, υ_{soil} is the Poisson's ratio of the soil, and ρ_0 is a displacement coefficient. Consider that the displacement coefficient can be evaluated as a function of the ratio $\chi = L_{slab}/B_{slab}$ using Figure 2.

For each case, compare the vertical displacement distributions of the equivalent pier (discretised in 200 elements in Thermo-Pile) with those characterising one of the piles of the group obtained through a previous analysis with reference to a single isolated situation. Comment on the impact of group effects on the vertical displacement distribution of an energy pile group compared to that of a single isolated energy pile under mechanical and/or thermal loads.

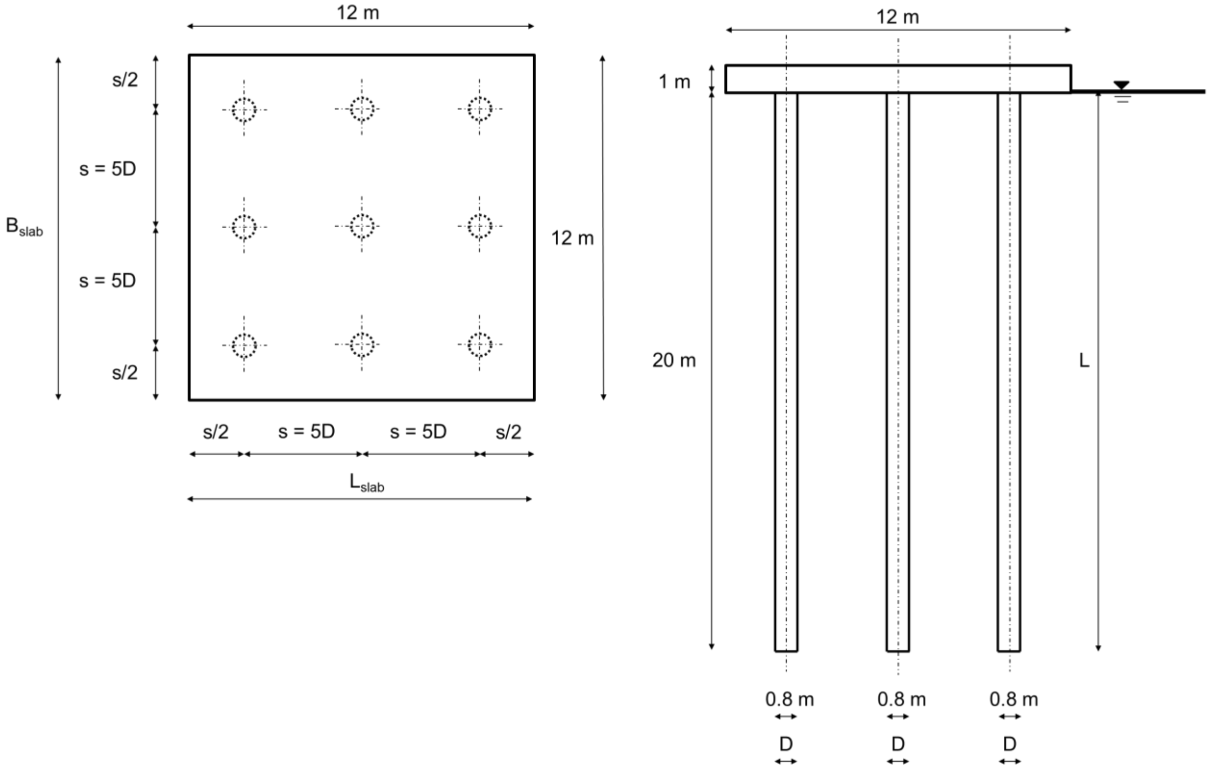

Figure 1. The problem.

Table 1. Sand properties.

	γ_{soil}	c'	φ'_{cv}	φ'	E_{soil}	ν_{soil}	α_r
	[kN/m^3]	[kPa]	[°]	[°]	[MPa]	[-]	[-]
Sand	19	20	31	38	78	0.3	0.33

Table 2. Pile properties.

	$\gamma_{concrete}$	E_{EP}	ν_{EP}	α_{EP}
	[kN/m^3]	[MPa]	[-]	[με/°C]
Pile	25	30000	0.25	10

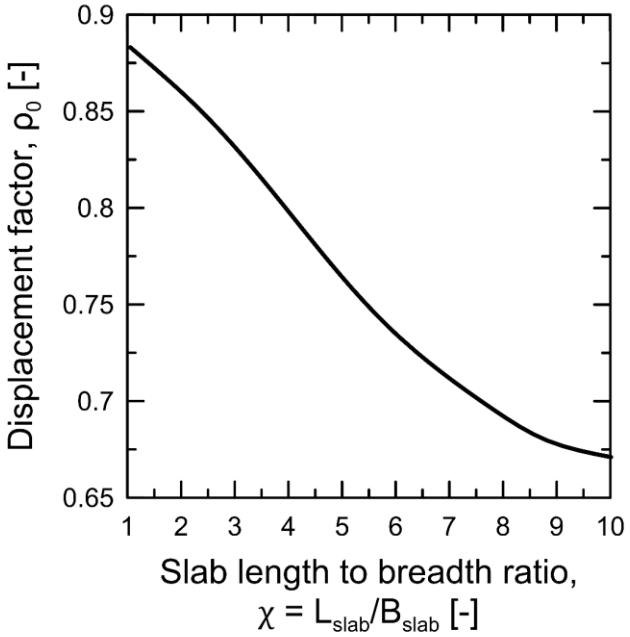

Figure 2. Displacement coefficient of a rigid rectangular plate resting on an isotropic elastic half-space (Gorbunov-Posadov and Serebrjanyi 1961).

Solution Part I

The equivalent diameter of the pier, D_{eq}, for a group of end-bearing piles can be evaluated as:

$$D_{eq} = \frac{2}{\sqrt{\pi}}\sqrt{A_g} = \frac{2}{\sqrt{\pi}}\sqrt{77.44} = 9.93 \text{ m}$$

where A_g is the plan area of the group and for a square group of energy piles is evaluated as:

$$A_g = [(\sqrt{n_{EP}} - 1)s + D]^2 = [(\sqrt{n_{EP}} - 1)5D + D]^2 = \left[\left(\sqrt{9} - 1\right) \cdot 5 \cdot 0.8 + 0.8\right]^2 = 77.44 \text{ m}^2$$

The homogenised Young's modulus of the equivalent pier, E_{eq}, can be estimated with the formulation proposed by Poulos (1993):

$$E_{eq} = \frac{A_{t,EP}\, E_{EP} + A_{soil}\, E_{soil}}{A_{t,EP}} = E_{EP}\, \frac{A_{t,EP}}{A_g} + E_{soil}\left(1 - \frac{A_{t,EP}}{A_g}\right) =$$
$$= 30000 \cdot \frac{4.52}{77.44} + 78 \cdot \left(1 - \frac{4.52}{77.44}\right) = 1751.03 + 73.45 = 1824.48 \text{ MPa}$$

$$A_{t,EP} = \pi \frac{D^2}{4} n_{EP} = \pi \cdot \frac{0.8^2}{4} \cdot 9 = 4.52 \text{ m}^2$$

For the evaluation of the equivalent linear thermal expansion coefficient of the pier, α_{eq}, it is assumed that $X = \alpha_{soil}/\alpha_{EP} \leq 1$. Hence:

$$\alpha_{eq} = \alpha_{EP}$$

The shaft and base resistances of the group read

$$q_{s,eq} = q_s \frac{D}{D_{eq}} n_{EP} = 18.36 \cdot \frac{0.8}{9.93} \cdot 9 = 13.31 \text{ kPa}$$

$$q_{b,eq} = q_b \frac{D^2}{D_{eq}^2} n_{EP} = 5302.44 \cdot \frac{0.8^2}{9.93^2} \cdot 9 = 309.74 \text{ kPa}$$

The slopes of the load-transfer relationships that govern the interaction between the shaft and base of the group with the surrounding soil read

$$K_{s,eq} = 0.8 \frac{E_M}{D} \zeta = 0.8 \frac{E_M}{D} \frac{s}{L} = 0.8 \frac{E_M}{D} \frac{5D}{L} = 0.8 \cdot \frac{34650}{0.8} \cdot \frac{5 \cdot 0.8}{20} = 6930 \text{ kPa/m}$$

$$K_{b,eq} = 4.8 \frac{E_M}{D} \zeta = 4.8 \frac{E_M}{D} \frac{s}{L} = 4.8 \frac{E_M}{D} \frac{5D}{L} = 4.8 \cdot \frac{34650}{0.8} \cdot \frac{5 \cdot 0.8}{20} = 41580 \text{ kPa/m}$$

To evaluate the slab stiffness, the following equation can be used:

$$K_{slab} = \frac{E_S \sqrt{B_{slab} L_{slab}}}{(1-{v_{soil}}^2)\rho_0} = \frac{78000 \cdot \sqrt{12 \cdot 12}}{(1-0.3^2) \cdot 0.88} = 1168831 \text{ kN/m}$$

Hence, the stiffness of the slab per unit cross-sectional area of equivalent pier is:

$$K_{slab}^* = \frac{K_{slab}}{A_g} = \frac{1168831}{77.44} = 15093 \text{ kPa/m}$$

The results obtained with the Thermo-Pile software are reported below.

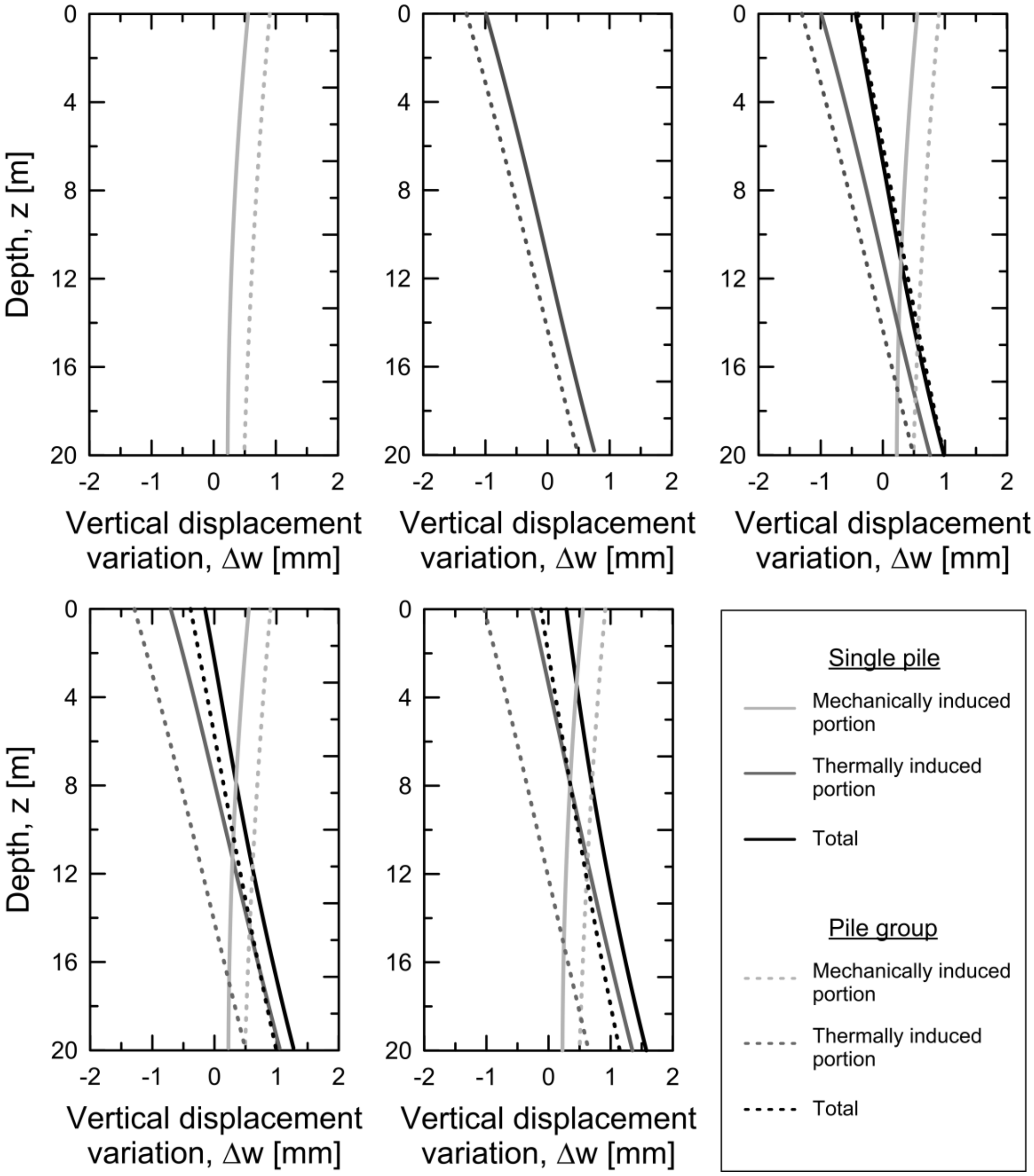

Figure 3. Results for CASES 1-4.

Solution Part II

The total axial displacement at the head of the pile in the isolated case was $w_i = 0.998\ mm$
With reference to the geometrical configuration two spacing need to be considered $s_1 = 3m$ and $s_2 = 4.24\ m$.
The parameters of interest for the definition of the interaction factors through the design charts are:

$$\frac{L}{D} = \frac{20}{0.8} = 25$$

$$\Lambda = \frac{E_{EP}}{G_{soil}} = \frac{30000}{30} = 1000$$

$$\frac{s_1}{D} = \frac{3}{0.8} = 3.75$$
$$\frac{s_2}{D} = \frac{4.24}{0.8} = 5.3$$

The obtained values of the interaction factor are:

$$\Omega_{s1} = 0.08$$

$$\Omega_{s2} = 0.06$$

The average head displacement is:

$$w_k = w_{ave} = w_i + 2w_i\Omega_{s1} + w_i\Omega_{s2} = 1.22 \text{ mm}$$

References

Frank, R., Kalteziotis, N., Bustamante, M., Christoulas, S., and Zervogiannis, H. 1991. Evaluation of performance of two piles using pressuremeter method. Journal of geotechnical engineering **117**(5): 695-713.

Gorbunov-Posadov, M.I., and Serebrjanyi, R.V. 1961. Design of structures on elastic foundation. *In* 5th International conference on Soil Mechanics and Foundation Engineering. pp. 643-648.

Knellwolf, C., Peron, H., and Laloui, L. 2011. Geotechnical analysis of heat exchanger piles. Journal of Geotechnical and Geoenvironmental Engineering **137**(10): 890-902.